Matched Filters for Binary Signals: A Correction and Elaboration

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This article is a correction and elaboration of "Matched Filters for Binary Signals," appearing in The Deep Space Network, Technical Report 32-1526, Vol. I, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1971.

A preceding article (Ref. 1) contains two element value tables (Tables 2, 3) which were meant to cover network orders 1 to 7. Due to a regrettable error, the tables actually incorporated in Ref. 1 are valid only up to order 5 (as evidenced by the negative element values for orders 6, 7). A version of these tables valid up to order 7 is presented here.

Strictly speaking, the networks described here differ from those of Ref. 1. With the adopted number of digits, these differences are too small to be evident in the case of orders 1 and 2. However, they do increase with the order. Thus, we find in the networks of order 5 a maximal discrepancy of 28% (in C_2). In spite of that, the difference in performance as measured by ν_N/ν_∞ of the actual networks (see Table 1 and footnote on p. 58 of Ref. 1) is so small that Table 1 and Fig. 4 of Ref. 1 are applicable, without modification, to the networks of Ref. 1 up to order 5 as well as to all orders of the networks presented here.

Another modification relates to the transmission-zero frequencies $FR(I)$. In the tables presented here, $FR(I)$ is the zero realized by section I and is thus the parallel resonance frequency of $L(I)$, $CR(I)$. (In the original tables, $FR(I)$ of the Nth order network referred to section $N + 2 - I$.)

We conclude with a brief discussion of the relationship of the two sets of networks. It was pointed out on p. 58 of Ref. 1 that, given the set of transmission zeros, the desired matched filter could be synthesized through a ladder development of any of the following immittances: $Z_{11}(s)$, $Z_{22}(s)$, $Y_{11}(s)$, $Y_{22}(s)$. (However, see footnote 2 on p. 58 of Ref. 1.)

With infinite precision, all four expansions would have yielded the same network. With finite precision in a real computer, differences were bound to show up yielding four different networks (for each order). Which of the four should one adopt? In considering this question we initially reasoned that since the ladder elements are computed in sequence, one would expect the errors in the element values to increase as the synthesis progressed. This suggested adopting the elements of the input half of the filter from $Z_{11}(s)$ or $Y_{11}(s)$ and the ouput half from $Y_{22}(s)$ or $Z_{22}(s)$.

Actual synthesis of all four families has revealed the presence of additional error mechanisms, leading to significant differences among the four families. The most striking difference is shown in the breakdown of the synthesis process. As networks of increasing order are synthesized, a point is eventually reached where the accumulated errors finally lead to negative elements and a breakdown of the process. This aspect of the four expansions is summarized in Table 4 of this article. Note the marked advantage of the $Z_{11}(s)$ expansion. Examination of Eq. (64) of Ref. 1 provides a clue to this behavior. $Z_{11}(s)$ is the only immittance whose construction from the polynomials $g(s), h(s)$ involves no "cancellation" errors, as it calls for the addition of positive numbers only. (The coefficients of $g_e(s)$, $g_o(s)$, $h_e(s)$ are all positive. Those of $h_o(s)$ are all negative.)

Obviously, the $Z_{11}(s)$ synthesis is the one to be preferred. Though it is valid up to order 8, we have decided to aim at higher precision and have restricted it to order 7. Tables 2 and 3 of this article (and originally meant for Ref. 1) are based on the $Z_{11}(s)$ expansion. On the other hand, the tables mistakenly incorporated in Ref. 1 are those based on the $Y_{22}(s)$ synthesis.

It should be stressed that the above arguments provide only a crude estimate of the reliability of the element values. As indicated in Section V of Ref. 1, a precise reliability indication was obtained through a direct and almost error-free analysis of the performance of the networks with the computed elements. In the present case, this approach turned out to be much simpler than the alternative of special synthesis techniques based on frequency transformations.

Reference

1. Zohar, S., "Matched Filters for Binary Signals," in *The Deep Space Network*, Technical Report 32-1526, Vol. I, pp. 52–62. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1971.

Table 2. Network elements for $T = \pi$ sec, $R_1 = 1\,\Omega$

UNITS   SECOND,HERTZ,OHM,HENRY,FARAD
T= 3.1415927+00
RGEN= 1.000+00

NET. ORDER	1	2	3	4	5	6	7
RLOAD	1.500+00	1.000+00	1.000+00	1.000+00	1.000+00	1.000+00	1.00n+00
SAMPLE GAIN	6.170-01	5.076-01	5.051-01	5.039-01	5.031-01	5.026-01	5.016-01
L (1)	1.410+00	5.006-01	2.997-01	2.158-01	1.689-01	1.389-01	1.176-01
CR(1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C (1)	1.182+00	8.890-02	2.270-02	1.030-02	5.878-03	3.813-03	2.373-03
L (2)		4.908-01	7.718-02	2.866-02	1.436-02	8.429-03	5.425-03
CR(2)		4.075-01	6.920-01	8.575-01	9.837-01	1.088+00	1.187+00
FR(2)		3.559-01	6.886-01	1.015+00	1.339+00	1.662+00	1.983+00
C (2)		1.749+00	6.944-02	1.312-02	4.669-03	2.114-03	2.074-03
L (3)			4.672-01	8.516-02	3.323-02	1.708-02	1.047-02
CR(3)			4.642-01	6.575-01	7.636-01	8.469-01	8.929-01
FR(3)			3.418-01	6.726-01	9.991-01	1.323+00	1.646+00
C (3)			1.995+00	7.014-02	1.318-02	4.689-03	2.431-03
L (4)				4.597-01	8.804-02	3.521-02	1.872-02
CR(4)				4.897-01	6.517-01	7.335-01	7.831-01
FR(4)				3.354-01	6.645-01	9.903-01	1.314+00
C (4)				2.100+00	7.061-02	1.326-02	4.324-03
L (5)					4.557-01	8.939-02	3.641-02
CR(5)					5.048-01	6.517-01	7.177-01
FR(5)					3.318-01	6.594-01	9.846-01
C (5)					2.160+00	7.095-02	1.309-02
L (6)						4.531-01	9.006-02
CR(6)						5.150-01	6.537-01
FR(6)						3.295-01	6.559-01
C (6)						2.198+00	7.110-02
L (7)							4.518-01
CR(7)							5.217-01
FR(7)							3.278-01
C (7)							2.233+00

Table 3. Network elements for $T = 10\ \mu s$, $R_1 = 50\ \Omega$

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***UNITS***    MICROSECOND,MEGAHERTZ,OHM,MICROHENRY,MICROFARAD
T= 1.0000000+01
RGEN= 5.000+01
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NET. ORDER	1	2	3	4	5	6	7
RLOAD	7.500+01	5.000+01	5.000+01	5.000+01	5.000+01	5.000+01	5.000+01
SAMPLE GAIN	6.170-01	5.076-01	5.051-01	5.039-01	5.031-01	5.026-01	5.016-01
L (1)	2.244+02	7.967+01	4.769+01	3.435+01	2.689+01	2.211+01	1.871+01
CR(1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C (1)	7.526-02	5.660-03	1.445-03	6.557-04	3.742-04	2.427-04	1.510-04
L (2)		7.812+01	1.228+01	4.561+00	2.285+00	1.341+00	8.633-01
CR(2)		2.594-02	4.406-02	5.459-02	6.262-02	6.928-02	7.557-02
FR(2)		1.118-01	2.163-01	3.190-01	4.208-01	5.221-01	6.231-01
C (2)		1.114-01	4.421-03	8.350-04	2.973-04	1.346-04	1.320-04
L (3)			7.435+01	1.355+01	5.289+00	2.718+00	1.666+00
CR(3)			2.955-02	4.186-02	4.861-02	5.391-02	5.684-02
FR(3)			1.074-01	2.113-01	3.139-01	4.158-01	5.172-01
C (3)			1.270-01	4.465-03	8.391-04	2.985-04	1.548-04
L (4)				7.316+01	1.401+01	5.604+00	2.980+00
CR(4)				3.117-02	4.149-02	4.670-02	4.985-02
FR(4)				1.054-01	2.087-01	3.111-01	4.129-01
C (4)				1.337-01	4.495-03	8.441-04	2.753-04
L (5)					7.252+01	1.423+01	5.795+00
CR(5)					3.214-02	4.149-02	4.569-02
FR(5)					1.042-01	2.072-01	3.093-01
C (5)					1.375-01	4.517-03	8.334-04
L (6)						7.211+01	1.433+01
CR(6)						3.279-02	4.162-02
FR(6)						1.035-01	2.061-01
C (6)						1.400-01	4.526-03
L (7)							7.190+01
CR(7)							3.321-02
FR(7)							1.030-01
C (7)							1.421-01

Table 4. Breakdown behavior

Immittance used for ladder expansion	Network order in which negative elements first appear
$Z_{11}(s)$	9
$Z_{22}(s)$	5
$Y_{11}(s)$	7
$Y_{22}(s)$	6